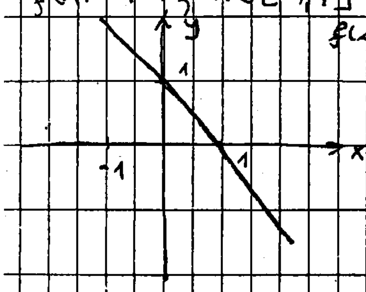
~ ПРИМЕРИ ТЕСТОВА ~

~ Група А ~

1. ЗАДАТАК

$f(x) = 1 - x, \quad x \in [-1, 1]$



$f(1) = 1$

$f(-1) = -1 = -f(1)$

$$a_0 = \frac{2}{1-(-1)} \left(\int_{-1}^{1} (1-x)\,dx \right) = \frac{2}{1+1} \left(\int_{-1}^{1} dx - \int_{-1}^{1} x\,dx \right) =$$

$$= \frac{2}{2} \left(x \Big|_{-1}^{1} - \frac{x^2}{2} \Big|_{-1}^{1} \right) = 1 \left((1-(-1)) - \left(\frac{1}{2} - \frac{1}{2} \right) \right) =$$

$$= 2$$

$$a_n = \frac{2}{2} \int_{-1}^{1} (1-x) \cos n\pi x\,dx = \int_{-1}^{1} \cos n\pi x\,dx - \int_{-1}^{1} x \cos n\pi x$$

$$= 2 \int_0^1 \cos n\pi x\,dx - \left(\cancel{\frac{1}{n\pi} x \sin n\pi x} + \frac{1}{n^2\pi^2} \cos n\pi x \right) \Big|_{-1}^{1} =$$

$$= 2 / n\pi \, \cancel{\sin n\pi x} \Big|_0^1 - \left(\frac{1}{n^2\pi^2} \cos n\pi x - \frac{1}{n^2\pi^2} \cos n\pi x \right) = 0$$

$$b_n = \frac{2}{2} \int_{-1}^{1} (1-x) \sin n\pi x\,dx = \cancel{\int_{-1}^{1} \sin n\pi x\,dx} - \int_{-1}^{1} x \sin n\pi x\,dx =$$

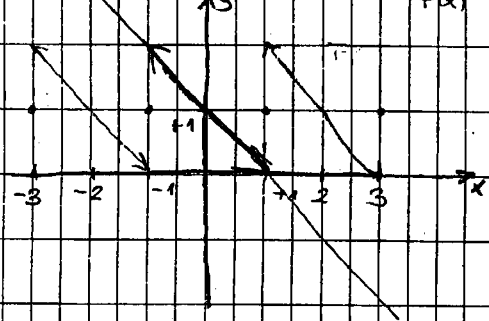
$$= -2 \int_0^1 x \sin n\pi x\,dx = -2 \left(-x / n\pi \cos n\pi x + \frac{1}{n^2\pi^2} \cancel{\sin n\pi x} \right) \Big|_0^1 =$$

непарна, непарна, парна

$$= \frac{2x}{n\pi} \cos n\pi x \Big|_0^1 = \frac{2}{n\pi} \cos n\pi = \frac{2}{n\pi} (-1)^n$$

$$\Phi(x) = 1 + \sum_{n=1}^{\infty} \frac{2(-1)^n}{n\pi} \sin n\pi x \qquad \Phi(x) = \begin{cases} f(x), & x \in (-1, 1) \\ 1, & x = -1,\ x = 1 \end{cases} \qquad T = 2$$

$\Phi(x)$



$$\frac{f(-1+) + f(1-)}{2} = \frac{1-(-1+) + 1-(1-)}{2} = \frac{2+0}{2} = \frac{2}{2} = 1$$

2. ЗАДАТАК

$x^2 y' = \sqrt{x} + 1, \quad y(1) = -3$

$x^2 \frac{dy}{dx} = \sqrt{x} + 1 \,/ \cdot \frac{dx}{x^2}, \quad x \neq 0$

$dy = \left(\frac{\sqrt{x}}{x^2} + \frac{1}{x^2} \right) dx$

$\int dy = \int (x^{1/2-2} + x^{-2})\,dx + C$

$y = \int (x^{-3/2} + x^{-2})\,dx + C$

$y = \int x^{-3/2} dx + \int x^{-2} dx + C$

$y = -2x^{-1/2} - x^{-1} + C$

$y = -2/\sqrt{x} - 1/x + C$

Ако $x = 0$; $0 \neq 1$ није решење

$-3 = -2/\sqrt{1} - 1/1 + C$

$-3 = -2/1 - 1 + C$

$-3 + 1 = -2 + C$

$-\cancel{2} = -\cancel{2} + C$

$C = 0$

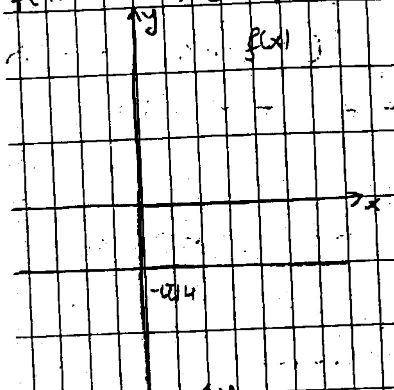
$y = -\frac{2}{\sqrt{x}} - \frac{1}{x}$ – пар. партикуларно решење

I ЗАДАТАК. ~ Група Б ~

1. ЗАДАТАК

$f(x) = -\pi/4 \quad x \in [0, \pi]$

$a_0 = \frac{2}{\pi} \int_0^{\pi} -\pi/4\,dx = \frac{2}{\pi}\left(-\frac{\pi}{4}\right) x \Big|_0^{\pi} = -\frac{1}{2}(\pi - 0) = -\pi/2$

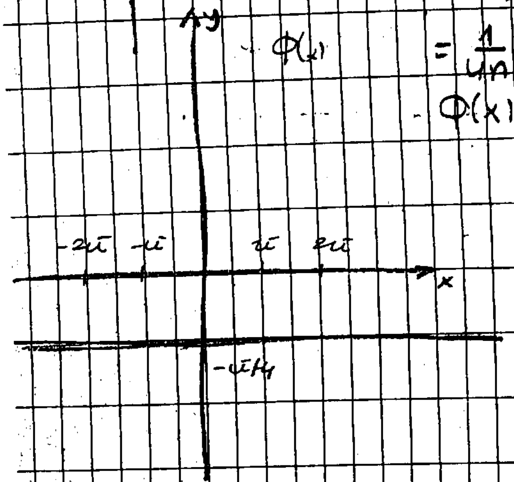


$a_n = \frac{2}{\pi} \int_0^{\pi} (-\pi/4) \cos 2nx\,dx = \frac{2}{\pi}\left(-\frac{\pi}{4}\right) \int_0^{\pi} \cos 2nx\,dx =$

$= -1/2 \cdot \frac{1}{2n} \sin 2nx \Big|_0^{\pi} = 0$

$b_n = \frac{2}{\pi} \int_0^{\pi} (-\pi/4) \sin 2nx\,dx = \frac{2}{\pi}\left(-\frac{\pi}{4}\right) \int_0^{\pi} \sin 2nx\,dx =$

$= -\frac{1}{2} \cdot \frac{-1}{2n} \cos 2nx \Big|_0^{\pi} = \frac{1}{4n} \cos 2n \Big|_0^{\pi} = \frac{1}{4n}(\cos 2n\pi - \cos 0) =$



$= \frac{1}{4n}(1 - 1) = 0$

$\Phi(x) = -\frac{\pi}{4}$

$\Phi(x) = \begin{cases} f(x), \forall x \in R \\ T = \pi \end{cases}$

* НАПОМЕНА: ОВО ЈЕ ЗА Фуријеов ред

sinusni Furijeov red *

2. ЗАДАТАК

$\underbrace{\left(\sqrt{y} - \frac{y}{x^2}\right)}_{P} dx + \underbrace{\left(\frac{x}{2\sqrt{y}} + \frac{1}{x}\right)}_{Q} dy = 0$

$u(x,y) = \sqrt{y}\,x + y/x + \varphi(y)$

$u'_y = \frac{x}{2\sqrt{y}} + \frac{1}{x} + \varphi'(y) = \frac{x}{2\sqrt{y}} + \frac{1}{x} \qquad \varphi'(y) = 0$

$P_y = \frac{1}{2\sqrt{y}} - 1/x^2 \qquad x^{-2} \quad x^{-1}$

$=$

$\varphi(y) = const. = C$

$u(x,y) = \sqrt{y}\,x + \frac{y}{x} + C = 0$

$Q_x = \frac{1}{2\sqrt{y}} - 1/x^2$

* $a_0 = 0 \quad a_n = 0 \quad n = 1, 2, \ldots, n \quad n \in N$

$b_n = \frac{2}{\pi} \int_0^{\pi} (-\pi/4) \sin 2nx\,dx = \frac{2}{\pi} \cdot 2 \cdot \left(-\frac{\pi}{4}\right) \int_0^{\pi} \sin 2nx\,dx =$

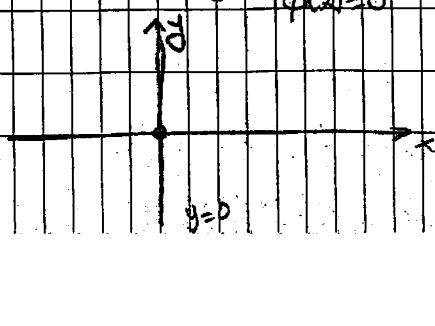
$= -\left(-\frac{1}{2n} \cos 2nx\right)\Big|_0^{\pi} =$

$g(x) = \begin{cases} f(x), & x \in [0, \pi] \\ 0, & x = 0 \\ -f(-x), & x \in [-\pi, 0) \end{cases}$

$= \frac{1}{2n} \cos 2nx \Big|_0^{\pi} = \frac{1}{2n}(\cos 2n\pi - \cos 0) = 0$

$b_n = 0$

$\Phi(x) = 0 \qquad \phi(x) = 0$



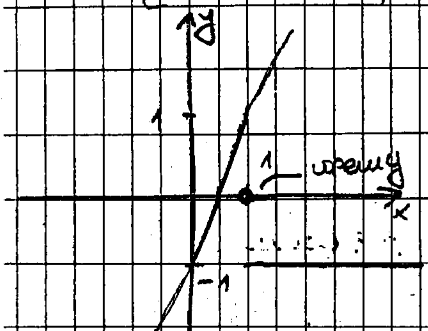
2

~ Група 4б ~

1. ЗАДАТАК

$f(x) = x - |x-1| \quad x \in [0,2]$

$$f(x) = \begin{cases} x-x-1, & x \geq 1 \quad , -1, \; x \geq 1 \\ x+x-1, & x < 1 \quad , 2x-1, \; x<1 \end{cases}$$



$$g(x) = \begin{cases} f(x), & x \in [0,1) \cup (1,2] \\ f(-x), & x \in [-2,0] \end{cases}$$

$b_n = 0, \; n = 0,1,2,3,\dots,n \;\; \forall n \in N$

$$a_0 = \frac{2}{2\cdot 2-0} \cdot 2\int_0^2 g(x)\,dx = \frac{4}{2}\left(\int_0^1 (2x-1)\,dx + \int_1^2 (-1)\,dx\right) = 2\left((x^2 - x)\Big|_0^1 - x\Big|_1^2\right) =$$

$$= 2\left(\cancel{1} - \cancel{1} - (2-1)\right) = 2(-1) = -2$$

$$a_n = \frac{2}{2} \cdot 2\left(\int_0^1 (2x-1)\cos n\pi x\,dx + \int_1^2 (-\cos n\pi x)\,dx\right) =$$

$$= 2\left(2\int_0^1 x\cos n\pi x\,dx - \int_0^1 \cos n\pi x\,dx - \int_1^2 \cos n\pi x\,dx\right) = 2\left(\cancel{2\frac{x}{n\pi}\sin n\pi x\Big|_0^1} + \frac{2}{n^2\pi^2}\cos n\pi x\Big|_0^1 - \cancel{\sin n\pi x \frac{1}{n\pi}\Big|_0^1} - \cancel{\frac{1}{n\pi}\sin n\pi x\Big|_1^2}\right) = \frac{2}{n^2\pi^2}(\cos n\pi - 1) =$$

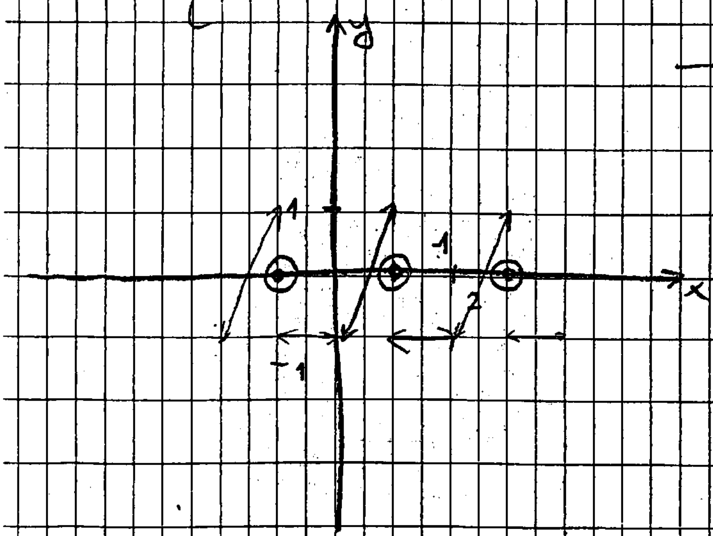
$$= \frac{2}{n^2\pi^2}\left((-1)^n - 1\right) = \begin{cases} a_{2k} = 0 \\ a_{2k+1} = \dfrac{-4}{(2k+1)^2\pi^2} \end{cases}$$

$$\Phi(x) = -1 - \frac{4}{\pi^2}\sum_{N=0}^{\infty} \frac{1}{(2N+1)^2} \cos(2N+1)\pi x$$

$$\Phi(x) = \begin{cases} g(x), & x \in [-2,0] \cup [0,1) \cup (1,2] \\ \end{cases}$$

$$\Phi(x) = \begin{cases} f(x), & x \in [0,1) \cup (1,2] \\ 0 & , \; x = 0, \; x = 1 \end{cases}$$

$$\frac{0 - (0-1) + 1 - |1-1|}{2} = \frac{-|-1| + 1}{2} = \frac{-1+1}{2} = \frac{0}{2} = 0$$



2. $(x+y)y'+y=x \quad | : (x+y)$

$$y'+\frac{y}{x+y}=\frac{x}{x+y}$$

$$y'+\frac{\cancel{y}}{\cancel{y}(\frac{x}{y}+1)}=\frac{\cancel{x}}{\cancel{x}(1+\frac{y}{x})}$$

$$y'+\frac{1}{\frac{x}{y}+1}=\frac{1}{\frac{y}{x}+1}$$

$u=\frac{y}{x} \qquad ux=y$

$y'=u'x+u$

$$u'x+u+\frac{1}{\frac{1}{u}+1}=\frac{1}{u+1}$$

$$u'x+u+\frac{u}{1+u}=\frac{1}{u+1}$$

$$u'x=\frac{1-u-u^2-u}{u+1}$$

$$u'x=\frac{-u^2-2u+1}{u+1}$$

$$\frac{du}{dx}x=\frac{-u^2-2u+1}{u+1} \qquad u\neq-1$$

$$\frac{du\cdot(u+1)}{-u^2-2u+1}=\frac{dx}{x}$$

$$\int\frac{(u+1)\,du}{-u^2-2u+1}=\int\frac{dx}{x}+c$$

$-2u-2=t \quad -2(u+1)du=dt$

$(u+1)du=-dt/2$

$$\int-\frac{dt}{2t}=\ln|x|+\ln(c)$$

$$-\frac{1}{2}\ln|t|=\ln|cx|$$

$$\ln\frac{1}{t^2}=\ln cx$$

$$\frac{1}{t^2}=cx$$

$$t^2=\frac{1}{cx}$$

$$(-2u-2)^2=\frac{1}{cx}$$

$$4(u+1)^2=\frac{1}{cx}$$

$$\left(\frac{y}{x}+1\right)^2=\frac{1}{4cx}$$

Ако $u=-1$, $u'=0$

$$0-1+\frac{1}{\frac{1}{-1}+1}=\frac{1}{-1+1}$$

$$-1+\frac{1}{-1+1}=\frac{1}{-1+1}$$

нису у $\mathcal{D}_f$ није решење

4

~ Група 2 ~

$x = x - 1$
$x = -x + 1$
$2x = 1$
$x = 1/2$

$- \quad | \quad +$
$\frac{1}{2}$

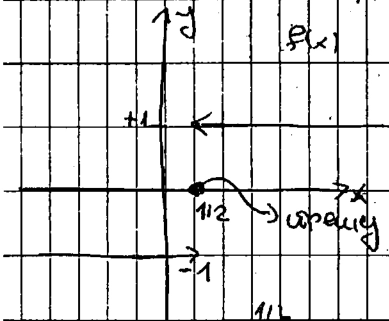
1. ЗАДАТАК

$f(x) = \mathrm{sgn}(x - |x-1|),\ x \in [0, 2]$

$$f(x) = \mathrm{sgn}\begin{cases} x - x + 1, & x > 1 \\ 1 - |1-1|, & x = 1 \\ x + x - 1, & x < 1 \end{cases} = \mathrm{sgn}\begin{cases} 1, & x > 1 \\ 1 - |0|, & x = 1 \\ 2x - 1, & x < 1 \end{cases} = \mathrm{sgn}\begin{cases} 1, & x > 1 \\ 1, & x = 1 \\ 2x - 1, & x < 1 \end{cases} = \begin{cases} 1, & x \geq 1 \\ 0, & x = 1/2 \end{cases}$$

$$= \begin{cases} +1, & x \in (1/2, \infty) \\ 0, & x = 1/2 \\ -1, & x \in (-\infty, 1/2) \end{cases}$$

$$a_0 = \frac{2}{2-0}\left(-\int_0^{1/2} dx + \int_{1/2}^{2} dx\right) = \frac{2}{2}\left(-x\Big|_0^{1/2} + x\Big|_{1/2}^{2}\right) = 1\left(-(1/2 - 0) + (2 - 1/2)\right) = (-1/2 + 3/2) = 1$$



$$a_n = \frac{2}{2}\left(-\int_0^{1/2} dx \cos n\pi x + \int_{1/2}^{2} \cos n\pi x\, dx\right) =$$
$$= \left(-\frac{1}{n\pi}\sin n\pi x\Big|_0^{1/2} + \frac{1}{n\pi}\sin n\pi x\Big|_{1/2}^{2}\right) =$$
$$= \left(-\frac{1}{n\pi}\sin\frac{n\pi}{2} - \frac{1}{n\pi}\sin\frac{n\pi}{2}\right) =$$
$$= -\frac{2}{n\pi}\sin\frac{n\pi}{2} = \begin{cases} a_{2k} = 0 \\ a_{2k+1} = -\frac{2}{(2k+1)\pi}\cdot(-1)^k \end{cases}$$

$$b_n = \frac{2}{2}\left(-\int_0^{1/2} \sin n\pi x\, dx + \int_{1/2}^{2} \sin n\pi x\, dx\right) =$$

$$= \left(\frac{1}{n\pi}\cos n\pi x\Big|_0^{1/2} - \frac{1}{n\pi}\cos n\pi x\Big|_{1/2}^{2}\right) = -\frac{1}{n\pi} - \frac{1}{n\pi} = -\frac{2}{n\pi}$$

$$\Phi(x) = \frac{1}{2} + \sum_{N=0}^{\infty}{}' \frac{-2}{(2N+1)\pi}(-1)^N \cos(2N+1)x + \sum_{N=1}^{\infty}{}' \frac{-2}{N\pi}\sin N\pi x$$

$$\underline{\underline{T = 2}}$$

$$\Phi(x) = \begin{cases} f(x), & x \in [0, 1/2) \cup (1/2, 2] \\ 0, & x = 1/2 \end{cases}$$

2. ЗАДАТАК

$y' + 1 = \sqrt[3]{x + y + 1},\ y(0) = 0$

$x + y + 1 = z(x)$

$z' = 1 + y' \qquad -y' = 1 - z'$

$\cancel{y' = 1 - z'} \qquad \underline{\underline{y' = z' - 1}}$

$z' - \cancel{1} + \cancel{1} = \sqrt[3]{z}$

$\frac{dz}{dx} = z^{1/3}$

$\frac{dz}{z^{1/3}} = dx \,\big|\, z \neq 0$

$\int z^{-1/3} dz = \int dx + C \qquad y(0) = 0$

$\frac{3}{2} \cdot z^{2/3} = x + C$

$\frac{3}{2} \cdot (x + y + 1)^{2/3} = x + C \qquad \frac{2}{3} C = C$

$\sqrt[3]{(x + y + 1)^2} = \frac{2}{3}x + C$

$(x + y + 1)^2 = \left(\frac{2}{3}x + C\right)^3$

$\sqrt[3]{(0 + 0 + 1)^2} = \frac{2}{3} \cdot 0 + C$

$\sqrt[3]{1^2} = 0 + C \qquad \sqrt[3]{1} = C \qquad C = 1$

$\sqrt[3]{(x + y + 1)^2} = \frac{2}{3}x + 1$

партикуларно решење

Ако $z = 0 \quad z' = 0 \quad -1 + 1 = \sqrt[3]{0}$

$x + y + 1 = 0 \qquad 0 \equiv 0 \quad T$

$y' = 0 - 1$

$y' = -1$

једно решење

$y = -x - 1$

~ Глава 5 Диференцијалне једначине вишег реда ~

5.1. Хомогена диференцијална једначина вишег реда са константним коефицијентима

Решити диференцијалну једначину:

1. $y''+y'-2y=0$

$\lambda^2+\lambda-2=0$

$\lambda_1=-2 \quad \lambda_2=1$

$y=C_1e^{-2x}+C_2e^{x}$

2. $y''-5y'+6y=0$

$\lambda^2-5\lambda+6=0$

$\lambda_1=2 \quad \lambda_2=3$

$y=C_1e^{2x}+C_2e^{3x}$

3. $y''+8y'+7y=0$

$\lambda^2+8\lambda+7=0$

$\lambda_1=-7 \quad \lambda_2=-1$

$y=C_1e^{-7x}+C_2e^{-x}$

4. $y''-6y'-40y=0$

$\lambda^2-6\lambda-40=0$

$\lambda_1=10 \quad \lambda_2=-4$

$y=C_1e^{-4x}+C_2e^{10x}$

5. $y''-2y'-3y=0$

$\lambda^2-2\lambda-3=0$

$\lambda_1=3 \quad \lambda_2=-1$

$y=C_1e^{3x}+C_2e^{-x}$

6. $y''+3y'+2y=0$

$\lambda^2+3\lambda+2=0$

$\lambda_1=-1 \quad \lambda_2=-2$

$y=C_1e^{-x}+C_2e^{-2x}$

7. $y''+y'=0$

$\lambda^2+\lambda=0$

$\lambda_1=0 \quad \lambda_2=-1$

$y=C_1+C_2e^{-x}$

8. $y''-2y'=0$

$\lambda^2-2\lambda=0$

$\lambda_1=0 \quad \lambda_2=2$

$y=C_1+C_2e^{2x}$

9. $y'+10y'=0$

$\lambda^2+10\lambda=0$

$\lambda_1=0 \quad \lambda_2=-10$

$y=C_1+C_2e^{-10x}$

10. $y''-y=0$

$\lambda^2-1=0$

$\lambda_1=-1 \quad \lambda_2=1$

$y=C_1e^{-x}+C_2e^{x}$

11. $y''-4y=0$

$\lambda^2-4=0$

$\lambda_1=-2 \quad \lambda_2=2$

$y=C_1e^{-2x}+C_2e^{2x}$

12. $y''-2y=0$

$\lambda^2-2=0$

$\lambda_1=-\sqrt{2} \quad \lambda_2=\sqrt{2}$

$y=C_1e^{-\sqrt{2}x}+C_2e^{\sqrt{2}x}$

13. $y''+2y'+y=0$

$\lambda^2+2\lambda+1=0$

$\lambda_1=-1=\lambda_2$

$y=C_1e^{-x}+C_2xe^{-x}$

14. $y''-6y'+9y=0$

$\lambda^2-6\lambda+9=0$

$\lambda_{1/2}=3$

$y=C_1e^{3x}+C_2xe^{3x}$

15. $y''+10y'+25y=0$

$\lambda^2+10\lambda+25=0$

$\lambda_{1/2}=-3$

$y=C_1e^{-5x}+C_2xe^{-5x}$

16. $y''+y'+y=0$

$\lambda^2+\lambda+1=0$

$\lambda_{1/2}=\frac{-1\pm\sqrt{1-4}}{2}$

$\lambda_{1/2}=\frac{-1\pm\sqrt{-3}}{2}$

$\lambda_{1/2}=\frac{-1\pm\sqrt{3}i}{2}$

$\lambda_{1/2}=-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$

$y=e^{-1/2}\cos\frac{\sqrt{3}}{2}x\cdot C_1+e^{-1/2}\sin\frac{\sqrt{3}}{2}x\cdot C_2$

17. $y''-y'+y=0$

$\lambda^2-\lambda+1=0$

$\lambda_{1/2}=\frac{1\pm\sqrt{1-4}}{2}$

$\lambda_{1/2}=\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$

$y=C_1e^{\frac{1}{2}x}\cos\frac{\sqrt{3}}{2}x+C_2e^{\frac{1}{2}x}\sin\frac{\sqrt{3}}{2}x$

18. $y''+2y'+2y=0$

$\lambda^2+2\lambda+2=0$

$\lambda_{1/2}=\frac{-2\pm\sqrt{4-8}}{2}$

$\lambda_{1/2}=\frac{-2\pm\sqrt{-4}}{2}$

$\lambda_{1/2}=\frac{-2\pm 2i}{2}$

$\lambda_{1/2}=-1\pm i$

$y=C_1e^{-x}\cos x+C_2e^{-x}\sin x$

19. $y''+y=0$

$\lambda^2+1=0$

$\lambda_{1/2}=\pm i$

$y=C_1\cos x+C_2\sin x$

20. $y''+2y=0$

$\lambda^2+2=0$

$\lambda_{1/2}=\pm\sqrt{2}i$

$y=C_1\cos\sqrt{2}x+C_2\sin\sqrt{2}x$

21. $y''+9y=0$

$\lambda^2+9=0$

$\lambda_{1/2}=\pm 3i$

$y=C_1\cos 3x+C_2\sin 3x$

22. $y'''+3y''+3y'+y=0$

$\frac{1\pm\sqrt{1-16}}{8}=\frac{1\pm\sqrt{-15}}{8}$

$\lambda^3+3\lambda^2+3\lambda+1=0$

$(\lambda^3+1)+3\lambda^2(\lambda+1)=0$

$(\lambda+1)(3\lambda^2+\lambda^2-\lambda+1)=0$

$(\lambda+1)(4\lambda^2-\lambda+1)=0$

$\lambda_1=-1\quad\lambda_2=\frac{1}{8}\pm\sqrt{15}/8\,i$

$y=C_1e^{-x}+C_2e^{1/8x}\cos\sqrt{15}/8\,x+C_3e^{1/8x}\sin\sqrt{15}/8\,x$

23. $y'''-y''=0$

$\lambda^3-\lambda^2=0$

$\lambda^2(\lambda-1)=0$

$\lambda_{1/2}=0\quad\lambda_3=1$

$y=C_1+C_2\cdot x+C_3e^x$

24. $y'''+2y''-y'-2y=0$

$\lambda^3+2\lambda^2-\lambda-2=0$

$\lambda^2(\lambda+2)-(\lambda+2)=0$

$(\lambda+2)(\lambda-1)(\lambda+1)=0$

$\lambda_1=-2\quad\lambda_2=1\quad\lambda_3=-1$

$y=C_1e^{-2x}+C_2e^x+C_3e^{-x}$

25. $y''' - 6y'' + 11y' - 6y = 0$

$$\begin{array}{c|c|c|c|c} 1 & 1 & -6 & 11 & -6 \\ \hline & 1 & -5 & 6 & 0 \end{array}$$

$\lambda^2 - 5\lambda + 6 = 0$

$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

$\lambda_1 = 1 \quad \lambda_2 = 2 \quad \lambda_3 = 3$

$y = C_1 e^{x} + C_2 e^{2x} + C_3 e^{3x}$

26. $y''' - 4y'' - 3y' + 18y = 0$

$$\begin{array}{c|c|c|c|c} -2 & 1 & -4 & -3 & 18 \\ \hline & 1 & -6 & 9 & 0 \end{array}$$

$\lambda^2 - 6\lambda + 9 = 0$

$\lambda^3 - 4\lambda^2 - 3\lambda + 18 = 0$

$\lambda_1 = -2 \quad \lambda_2 = 3 \quad \lambda_3 = 3$

$y = C_1 e^{-2x} + C_2 e^{3x} + C_3 x e^{3x}$

27. $y''' - 2y' = 0$

$y = C_1 + C_2 e^{\sqrt{2}x} + C_3 e^{-\sqrt{2}x}$

$\lambda^3 - 2\lambda = 0$

$\lambda_1 = 0 \quad \lambda_2 = \sqrt{2} \quad \lambda_3 = -\sqrt{2}$

28. $y''' - y = 0$

$\frac{-1 \pm \sqrt{1-4}}{2}$

$\lambda^3 - 1 = 0$

$y = C_1 e^{x} + C_2 e^{-1/2x} \cos \sqrt{3}/2x + C_3 e^{-1/2x} \sin \sqrt{3}/2x$

$(\lambda - 1)(\lambda^2 + \lambda + 1) = 0$

$\lambda_1 = 1 \quad \lambda_2 = -1/2 + \sqrt{3}/2 i \quad \lambda_3 = -1/2 - \sqrt{3}/2 i$

29. $y''' + y' = 0 \quad \lambda^2 + 1$

$\lambda^3 + \lambda = 0$

$y = C_1 + C_2 \cos x + C_3 \sin x$

$\lambda_1 = 0 \quad \lambda_2 = +i \quad \lambda_3 = -i$

30. $y''' + y = 0$

$\frac{1 \pm \sqrt{1-4}}{2}$

$\lambda^3 + 1 = 0$

$y = C_1 e^{-x} + C_2 e^{1/2x} \cos \sqrt{3}/2x + C_2 e^{1/2x} \sin \sqrt{3}/2x$

$(\lambda + 1)(\lambda^2 - \lambda + 1) = 0$

$\lambda_1 = -1 \quad \lambda_2 = 1/2 + \sqrt{3}/2 i \quad \lambda_3 = 1/2 - \sqrt{3}/2 i$

31. $y^{iv} + y''' = 0 \quad y = e^{rx},\ r = const.$

$R^5 e^{Rx} + R^3 e^{Rx} = 0$

$R^3 e^{Rx}(R^2 + 1) = 0$

$R_{1/2/3} = 0 \quad R_{4/5} = \pm i$

$y =$

$((e^{rx})')^{v} + y''' = 0$

$(e^{rix})^{v} + y''' = 0$

$(\cos r + i\sin r)^{(v)} + y''' = 0$

41. $y^{v} + y''' = 0$ $\quad y = C_1 + C_2 + C_3 + C_4 \cdot \cos x + C_5 \sin x$

$\lambda^{v} + \lambda^{3} = 0$

$\lambda^3(\lambda^{v-3} + 1) = 0$

$\lambda_{1,2,3} = 0 \quad v > 3$

$\lambda = -1 \leftarrow v = 3 + 2k + 1 \quad \lambda = \pm i$

12. y^{v}